3.4.2 Non-Ideal Three Phase Transformer Model

The non-ideal three phase transformer model can be derived from the proper interconnection of the non-ideal single phase transformers. For simplicity we assume that each single phase transformer is represented with its simplified non-ideal model. For the case of a delta-wye connected transformer, the result is illustrated in Figure 3.29.

For the circuit of Figure 3.29, the following relationships are valid:

$$\tilde{I}_a = (\tilde{V}_a - \tilde{E}_a)Y \qquad \tilde{I}_a^{'} = a\tilde{I}_a$$
$$\tilde{I}_b = (\tilde{V}_b - \tilde{E}_b)Y \qquad \tilde{I}_b^{'} = a\tilde{I}_b$$
$$\tilde{I}_c = (\tilde{V}_c - \tilde{E}_c)Y \qquad \tilde{I}_c^{'} = a\tilde{I}_c$$

$$\tilde{E}_a = a(\tilde{V}_A - \tilde{V}_B) \qquad \tilde{I}_A = \tilde{I}_c^{'} - \tilde{I}_a^{'}$$
$$\tilde{E}_b = a(\tilde{V}_B - \tilde{V}_C) \qquad \tilde{I}_B = \tilde{I}_a^{'} - \tilde{I}_b^{'}$$
$$\tilde{E}_c = a(\tilde{V}_C - \tilde{V}_A) \qquad \tilde{I}_C = \tilde{I}_b^{'} - \tilde{I}_c^{'}$$

Upon elimination of the variables $\tilde{E}_a, \tilde{E}_b, \tilde{E}_c$ and $\tilde{I}_a^{'}, \tilde{I}_b^{'}, \tilde{I}_c^{'}$ and expressing the remaining currents as a function of the voltages we obtain a set of six equations which, written in matrix notation, are:

$$\begin{bmatrix} \tilde{I}_a \\ \tilde{I}_b \\ \tilde{I}_c \\ \tilde{I}_A \\ \tilde{I}_B \\ \tilde{I}_C \end{bmatrix} = Y \begin{bmatrix} 1 & 0 & 0 & -a & a & 0 \\ 0 & 1 & 0 & 0 & -a & a \\ 0 & 0 & 1 & a & 0 & -a \\ -a & 0 & a & 2a^2 & -a^2 & -a^2 \\ a & -a & 0 & -a^2 & 2a^2 & -a^2 \\ 0 & a & -a & -a^2 & -a^2 & 2a^2 \end{bmatrix} \begin{bmatrix} \tilde{V}_a \\ \tilde{V}_b \\ \tilde{V}_c \\ \tilde{V}_A \\ \tilde{V}_B \\ \tilde{V}_C \end{bmatrix}$$

Note that above equation expresses the input/output relationship of the three phase transformer. In compact matrix form, above equation can be written as

$$\begin{bmatrix} \tilde{I}_{abc} \\ \tilde{I}_{ABC} \end{bmatrix} = Y \begin{bmatrix} I & -aE \\ -aE^T & a^2F \end{bmatrix} \begin{bmatrix} \tilde{V}_{abc} \\ \tilde{V}_{ABC} \end{bmatrix}$$

where I is the 3x3 identify matrix, and the matrices E and F are:

$$E = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Above equations represent the simplified model of a delta-wye connected three phase transformer. The same procedure can provide the models for other connections, i.e. delta-delta, wye-wye and wye-delta connections.

3.4.3 Sequence Circuits of Three Phase Transformers

Three phase transformers are inherently symmetric three phase elements. This means that by applying the symmetrical transformation, their model can be transformed to three equivalent circuits, namely the positive, negative and zero sequence equivalent circuits. The procedure will be illustrated on a delta-wye connected transformer model developed in the previous paragraph. It should be understood that the procedure equally applies to any other configuration.

The phase voltages and currents are substituted with their corresponding symmetrical components as follows:

$$\tilde{I}_{abc} = T\,\tilde{I}_{120}$$
$$\tilde{I}_{ABC} = T\,\tilde{I}_{120}^{'}$$
$$\tilde{V}_{abc} = T\,\tilde{V}_{120}$$
$$\tilde{V}_{ABC} = T\,\tilde{V}_{120}^{'}$$

Replacing the phase quantities with the symmetrical components, the equation for the three phase transformer becomes:

$$\begin{bmatrix} \tilde{I}_{120} \\ \tilde{I}_{120}^{'} \end{bmatrix} = Y \begin{bmatrix} T^{-1} I\,T & -a\,T^{-1} E\,T \\ -a\,T^{-1} E^{T} T & a^{2} T^{-1} F\,T \end{bmatrix} \begin{bmatrix} \tilde{V}_{120} \\ \tilde{V}_{120}^{'} \end{bmatrix}$$

Note that by direct evaluation, the following apply:

$$T^{-1} I\,T = I$$

$$T^{-1}ET = \begin{bmatrix} \sqrt{3}e^{j30^o} & 0 & 0 \\ 0 & \sqrt{3}e^{-j30^o} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$T^{-1}E^{T}T = \begin{bmatrix} \sqrt{3}e^{-j30^o} & 0 & 0 \\ 0 & \sqrt{3}e^{j30^o} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad T^{-1}FT = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Upon substitution and grouping the six equations into three groups of two we obtain:

$$\tilde{I}_1 = Y\tilde{V}_1 - \sqrt{3}aYe^{j30^o}\tilde{V}_1'$$
$$\tilde{I}_1' = -\sqrt{3}aYe^{-j30^o}\tilde{V}_1 + 3a^2Y\tilde{V}_1'$$

$$\tilde{I}_2 = Y\tilde{V}_2 - \sqrt{3}aYe^{-j30^o}\tilde{V}_2'$$
$$\tilde{I}_2' = -\sqrt{3}aYe^{j30^o}\tilde{V}_2 + 3a^2Y\tilde{V}_2'$$

$$\tilde{I}_0 = Y\,\tilde{V}_0,$$
$$\tilde{I}_0' = 0$$

Note that above relations represent three independent set of equations corresponding to three equivalent circuits which are illustrated in Figure 3.30. An example will illustrate the procedure.

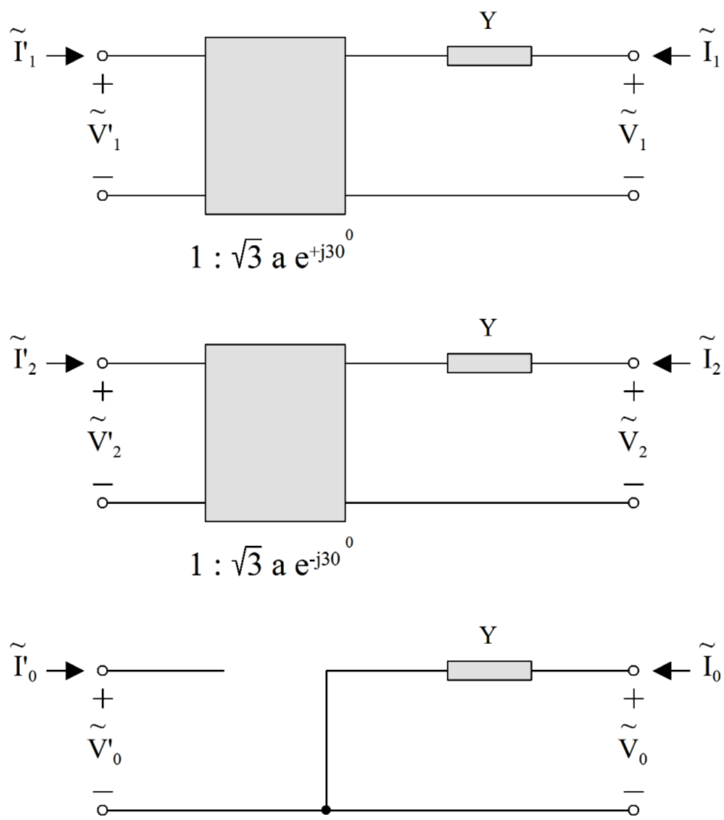
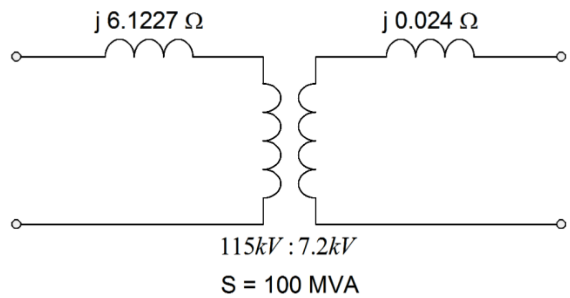


Figure 3.30: Sequence Equivalent of a Delta - Wye Connected Three Phase Transformer

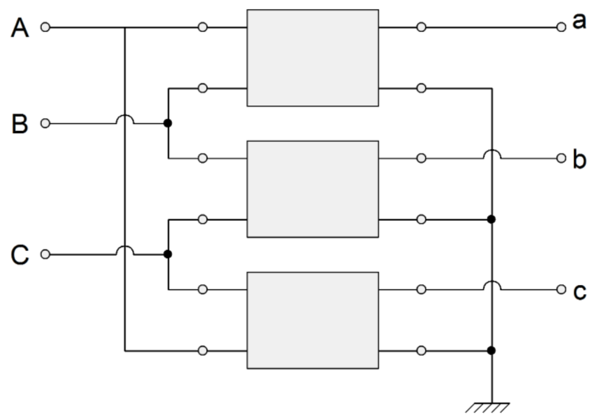
Example E3.7: A three phase transformer bank is made from three single-phase transformers. Each single phase transformer has the equivalent circuit of Figure E3.7a. The three phase connections are illustrated in Figure E3.7b.

a) Draw the positive sequence equivalent circuit of the three phase transformer bank with all impedances shown on the right hand side. The transformer ratio and the impedance values should be clearly marked in actual quantities, i.e. volts and ohms.
b) Draw the positive sequence equivalent circuit of the three phase transformer bank with all impedances shown on the left hand side. The transformer ratio and the impedance values should be clearly marked in actual quantities, i.e. volts and ohms.
c) Draw the per phase equivalent circuit in per unit of the three phase transformer bank using the following bases.

Left Hand Side:	S_B = 300 MVA (three phase) V_{B1} = 66.395 kV (line to neutral)
Right Hand Side:	S_B = 300 MVA (three phase) V_{B2} = 7.2 kV (line to neutral)



(a)



(b)

Figure E3.7: Construction of a Three Phase Transformer Bank
(a) Circuit Model of Single Phase Transformer, (b) Three Phase Connections. Each Block Represents the Single Phase Transformer of (a)

Solution: The equivalent circuit of Figure E3.7a can be modified by referring the j6.6125 ohm leakage impedance on the right hand side. By doing so, the three phase transformer model becomes identical to the circuit of Figure 3.11 with

$$Y = \frac{1}{j0.048\,ohms} = -j20.8333\,S \quad and \quad a = \frac{7.2}{115.0} = 0.0626$$

a) By utilizing the results of the previous section, the positive sequence equivalent circuit of the transformer is shown in Figure E3.7a.

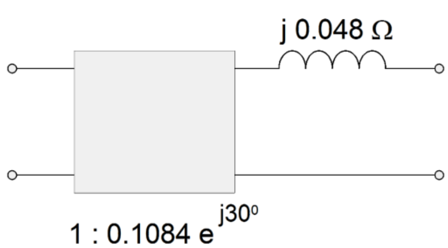


Figure E3.7a

b) By referring the impedance on the left hand side, the positive sequence equivalent circuit of Figure E3.7b is obtained.

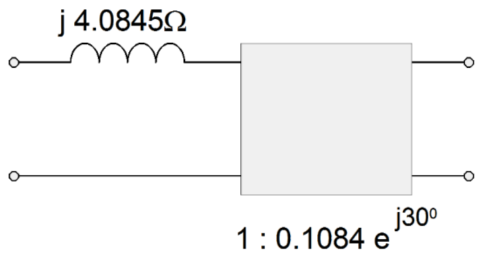


Figure E3.7b

c) The base impedance at the left hand side is:

$$Z_{b1} = \frac{V_{b1}^2}{S_{b1}} = \frac{115^2}{300} = 44.0833\,ohms$$

The per-unit value of the impedance is:

$$Z_{1u} = \frac{j4.0845}{44.0833} = j0.0927$$

The per unit transformation ratio is:

$$1:0.1084e^{j30^0} \rightarrow \quad 1/66.395:0.1084e^{j30^0}/7.2 \rightarrow \quad 1:1e^{j30^0}$$

Therefore the positive sequence per-unit equivalent circuit is shown in Figure E3.7c. Similarly the circuit of Figure E3.7d is developed.

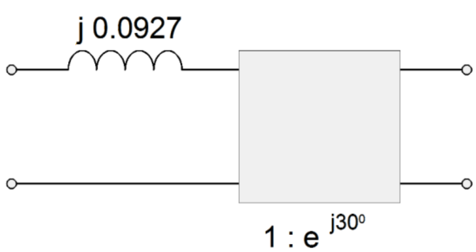


Figure E3.7c

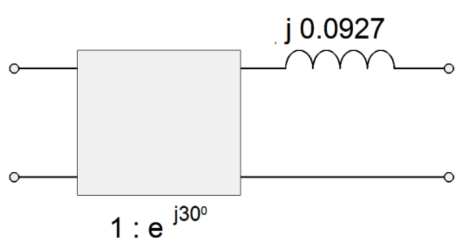


Figure E3.7d

3.4.4 Transformer Model for Inrush Current Computations

Transformers are normally made with iron core. Iron core transformers exhibit nonlinearities whenever the iron core is saturated. The iron core of a transformer can be represented with a nonlinear inductor and a nonlinear resistor.

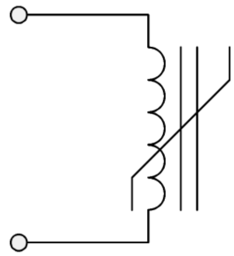


Figure 3.31: A Saturable Inductor Schematic Symbol

The equations describing a typical iron core represented with a saturable inductor are:

$$i_1(t) = i_0 \left| \frac{\lambda(t)}{\lambda_0} \right|^n sign(\lambda(t)) \tag{3.1}$$

$$i_2(t) = -i_0 \left| \frac{\lambda(t)}{\lambda_0} \right|^n sign(\lambda(t)) \tag{3.2}$$

$$0 = v_1(t) - v_2(t) - \frac{d\lambda(t)}{dt}$$

Above model provides the electric current required to support the magnetic flux of the transformer.

The core loss is a function of frequency and magnetic flux. Specifically, core loss consists of hysteresis loss and eddy current loss. Approximate equations for these losses are:

Hysteresis Loss:

$$P_h = a_h f B_{\max}^{\nu}, \quad \nu \approx 1.7$$

Eddy-Current Loss:

$$P_e = a_e f B_{\max}^2$$

where a_h *and* a_e are constants, $B_{\max}$ is the maximum magnetic flux density in the core.

3.4.5 Transformer Model for High Frequencies

At very high frequencies, a power transformer behaves quite differently. For example at high frequencies, the iron core of the transformer will not respond to the very fast changing field and it will not achieve full magnetization. In addition the core losses will be very high. Beyond a certain frequency, the response of the iron core will be non-existent to the point that the core can be neglected. In this case the transformer coil can be modeled as a distributed inductance and capacitance, the parameters of these components determined by the coil alone. Modeling of transformers at high frequencies is beyond the scope of this book.

3.5 Generator Modeling

Generators are very complex devices that exhibit a rather complex behavior. First generators do produce harmonics under normal operating conditions due to the construction of the windings. In addition, the generator exhibits different impedances along its direct and quadrature axes. This makes the interaction of the generator with the network rather complex when the waveforms are distorted.

Generator models for power frequency analysis can be classified into three categories depending on the time period of concern. For short duration phenomena (typically two cycles), the appropriate generator model is the so-called subtransient model. For longer duration periods (typically half second), the appropriate generator model is the transient model. Finally for long periods of time (or steady state), the appropriate model is the synchronous model. As an example Figure 3.32 illustrates the positive sequence transient model of the generator. Note that the generator exhibits a different value of impedance along the d-axis and a different along the q-axis.

A simplified way to model a generator in the presence of waveform distortion is to consider the generator impedances to the flow of specific harmonics. For this purpose it is expedient to analyze the electric currents into harmonics and each harmonic into sequence components.

The generator sequence impedances are:

$$L_h^+ \cong \frac{L_d^" + L_q^"}{2}$$

$$L_h^- \cong \frac{L_d^" + L_q^"}{2}$$

$$L_h^0 \cong L^0$$

It is important to recognize that harmonic currents in the armature of a synchronous machine will generate a magnetic flux in the air-gap of the machine. For example a positive sequence 5th harmonic current will generate a rotating magnetic field in the air-gap that will rotate at a speed equal five times the synchronous speed in the direction of the rotation of the rotor. Similarly, a negative sequence 5th harmonic current will generate a rotating magnetic field with speed equal five times the synchronous speed but at a direction opposite the rotation of the rotor. Finally a zero sequence 5th harmonic current will generate a pulsating magnetic field. These magnetic fields will generate a torque by the same mechanism as in an induction motor. This torque many times is harmful to the life expectancy of synchronous machines.

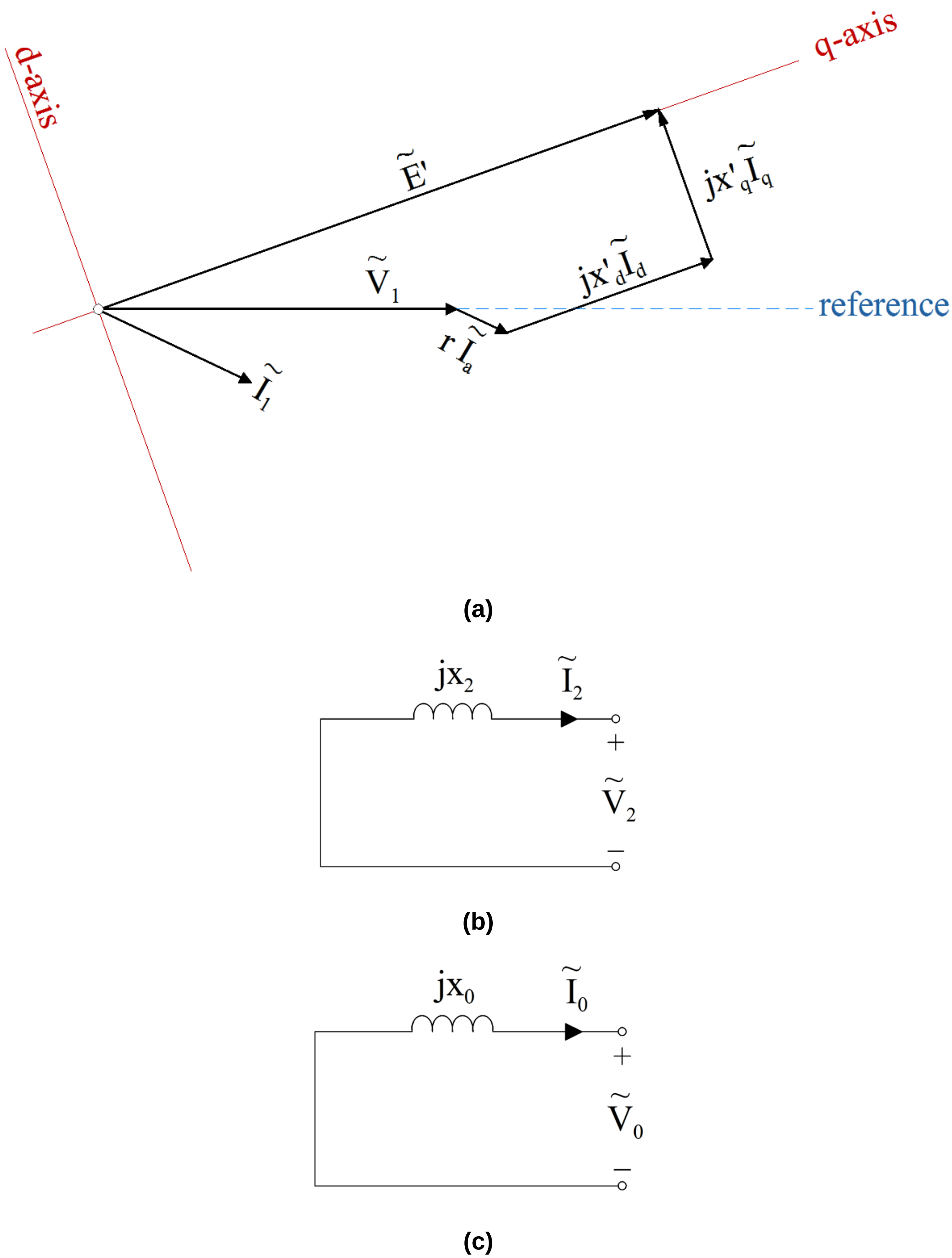


Figure 3.32: Synchronous Generator Transient Model – Phasor Diagram
(a) Positive sequence phasor diagram, (b) Negative sequence equivalent circuit, (c) Zero sequence equivalent circuit

3.6 Inverter Interfaced Generation Models

In the last couple of decades, we have experience the penetration of generation that is interfaced to the power grid via inverters. The most well-known systems are wind turbine systems and PV plants. The models of these systems are quite complex as they involve rotating machines and power electronics. As an example the figures below illustrate the most common wind turbine systems, referred to as type 3 and type 4. Figure 3.33 shows a wind turbine system that interfaces to the power grid through a rectifier that rectifies the power into DC and then it converts into AC of the proper frequency before connecting to the grid. The system has complete rectification and inversion of the full power generated by the wind turbine/generator. Figure 3.34 illustrates another system that has the wind system generator connected directly to the power grid and the frequency is controlled with a set of rectifier/inverter connected to the rotor winding of a double fed induction machine. As the speed of rotation of the wind turbine/generator system changes, the frequency of the rotor windings is changing in such a way that the armature currents and voltages are of the grip frequency.

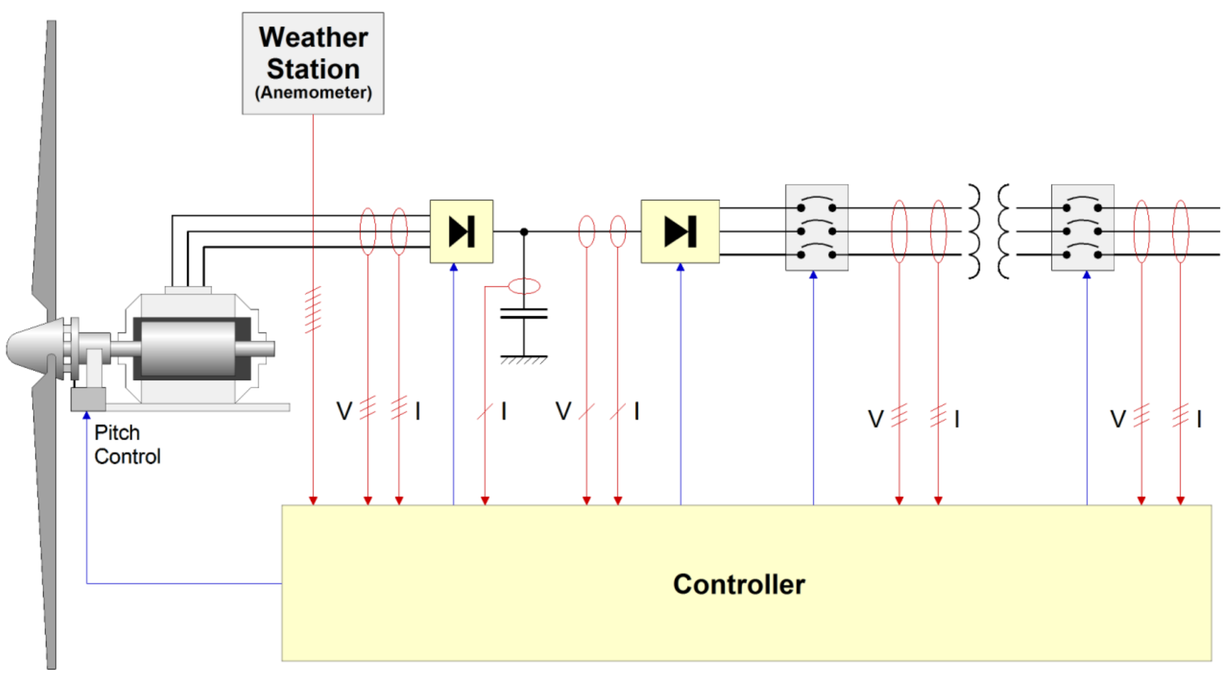


Figure 3.33: A Wind Turbine System with Full Rectification and Inversion

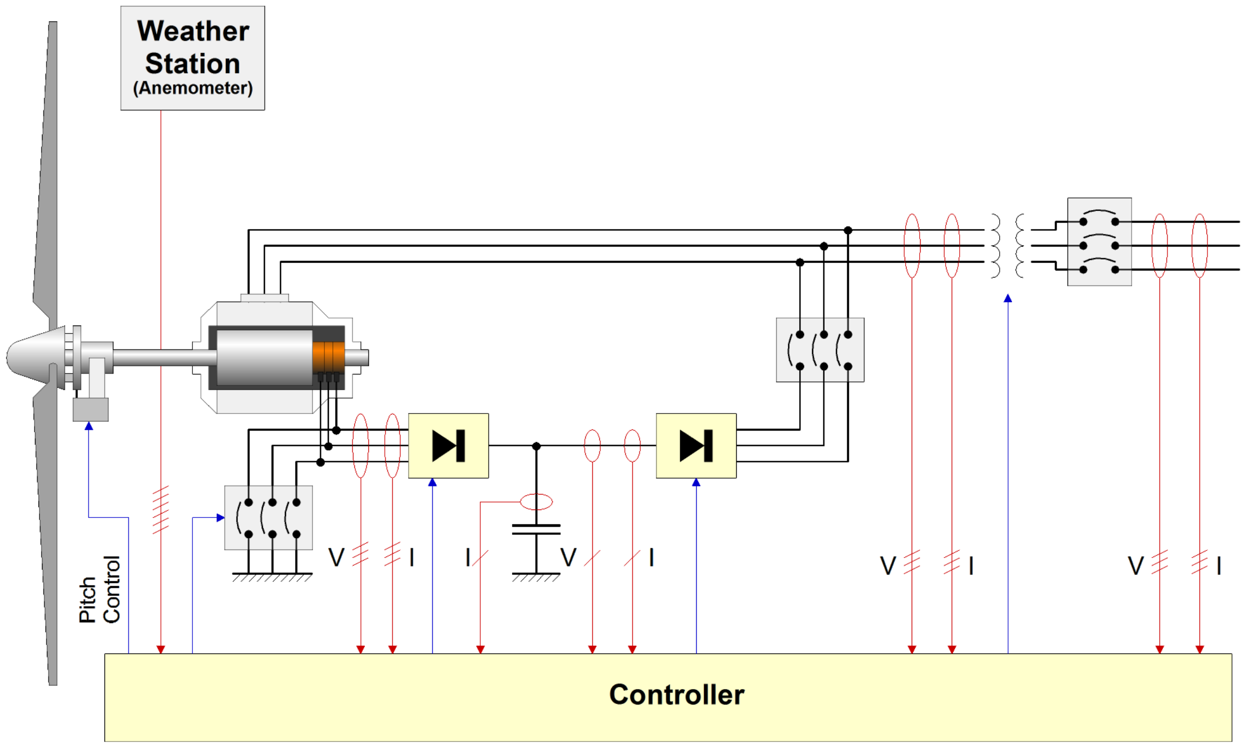


Figure 3.34: A Wind Turbine System with Double Fed Induction Machine and Rotor Winding Control

3.7 Electric Load Modeling

Electric loads have experienced a dramatic change in recent years. The reason for these changes is the proliferation of new technologies (especially power electronics) in the utilization of the electric energy. Electric loads consist of a variety of apparatus such as traditional induction motors, variable speed drives, computers, rectifiers, electronic ballast fluorescent lighting, dimmers, printers, air-conditioners, power conditioners, etc. Each of these load types has specific characteristics. In general, the load model to be selected depends on the intended application. For example, for traditional power flow applications, the load is modeled as a constant power load, or a constant current load or a constant impedance load or combination of these. There is some rational for each one of them. For example, a traditional feeder with many different loads on it will have a voltage regulator that maintains almost constant voltage at the start of the feeder while conditions may change elsewhere in the system. In this case the load of the feeder will be approximately constant for a specific set of customers connected to the feeder (no customer switching in and out). Similar arguments can be made for the constant current (specific loads) and constant impedance loads (typically no voltage regulation). Because a typical feeder may have a variety of loads, then occasionally a combination of these models is used, referred to as ZIP model (Z stands for constant impedance, I stands for constant current, and P stands for constant power). These models are approximations for steady state operation of the system. During transients, these models fail to represent the actual behavior of the system, even in the presence of voltage regulators (voltage regulators are typically electromechanical and have delayed response).

Here we present few examples, such as induction motors, dimmers, power supplies, fluorescent lighting, and converters.

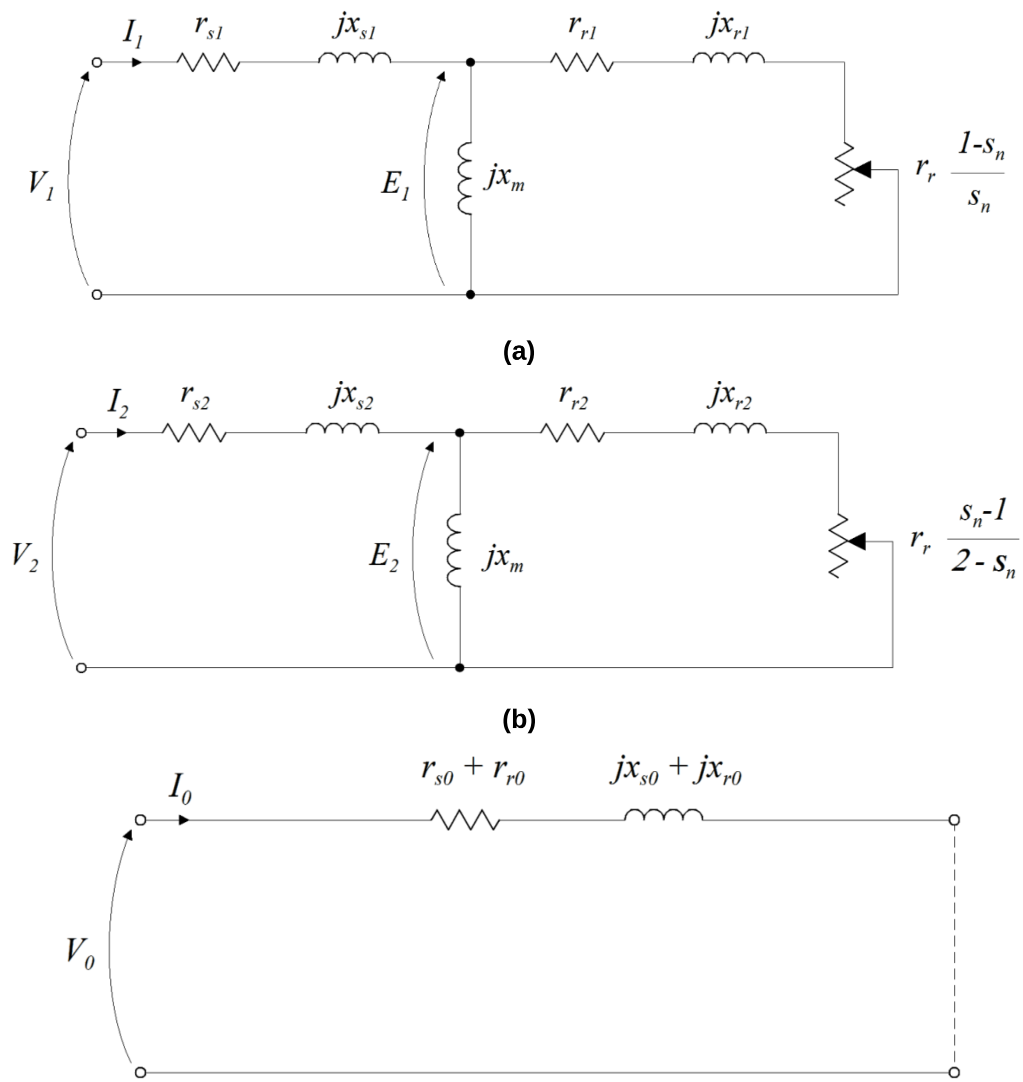
3.7.1 Induction Motors

The majority of electric loads are electric motors, typically two thirds of electric loads are electric motors. Electric motors are dominated by induction type motors. It is estimated that about 50% to 60% of the total electric load consists of induction type motors. Induction motors behave in a manner that affects the steady state operation of the system (typically the required reactive power depends on the voltage magnitude in a manner that if the voltage is reduced, the motor will require more reactive power that will cause further voltage reduction and possibly voltage instability). They also affect the transient response of the system as any transient may result in speed reduction of the motor which alters the characteristics of the motor. In this case the electromechanical transients become very important in determining the response of the system. In any case, electric motors do affect the stability of the system as well as protective relaying practices.

The issue of modeling the dynamics of the load, which by enlarge depend on electric motors, is an issue of intense research. In this section we examine the main characteristics of induction motors with emphasis on the impact on protection functions.

The most well-known induction motor designs are the NEMA designs A, B, C and D. Of course in recent years many new designs have been developed to the point where we have a large number of designs. For simplicity, we should discuss the standard designs A, B, C and D. An equivalent model that captures fairly accurately the characteristics of the standard designs is shown in Figure 3.x in terms of the positive, negative and zero sequence circuits of the induction motor (or generator).

In Figure 3.x the subscript s refers to stator, the subscript r refers to rotor and the subscripts 1, 2 and 0 refer to positive, negative and zero sequence respectively. In terms of energy conversion, the amount of energy consumed by the speed dependent resistors equals the amount of energy transformed from electrical into mechanical (or vice versa if the machine operates as a generator).



(c)

Figure 3.35 Sequence Model of Standard Induction Machine Designs.

(a) positive sequence, (b) negative sequence, (c) zero sequence

3.7.2 Inverter/Rectifier Modeling

One of the most common devices in power systems is the converter. The converter is an electronic switch based device that converts electric power from one form into another by switching operations. Converters can operate as rectifiers (AC to DC) or as inverters (DC to AC) or as general converters (AC to AC). The number of converter designs is enormous. For practical reasons we limit the discussion and presentation to one specific design. The reader is encouraged to consult books that are focused on modeling inverters.

For high power applications, a six valve converter design is often used. Figure 3.z illustrates the topology of the six valve converter. It consists of six switches implemented using SCRs. Each

SCR is protected by an R-C snubber circuit which reduces the transient voltage across each SCR during the transition from conducting to non-conducting state. A six-pulse converter operation is characterized by six switching operations per cycle. 12, 18, and 24 pulse converter topologies are obtained by connecting two, three or four six-valve converters in series with appropriate control signals and phase shifted A/C sources. Table 3.2 shows typical values of current waveform harmonics (in per-unit) generated by 6-pulse, 12-pulse, 18-pulse, and 24-pulse converters. Note that harmonic levels are decreasing with higher pulse topologies.

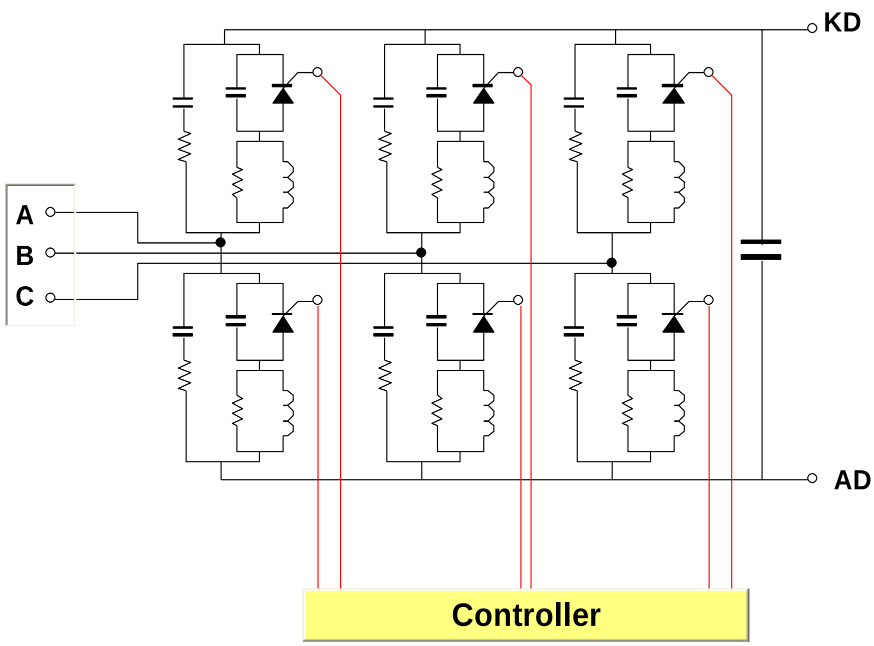


Figure 3.36: Generic Structure of a Converter

Averaging Model: For the averaging model of the two-level converter, we present equivalent equations of the two-level converter in phasor representation. The model equations are:

External-state equations are:

$$\tilde{I}_a = -jB(\tilde{V}_a - \tilde{E}_a) \quad (0.1)$$

$$\tilde{I}_b = -jB(\tilde{V}_b - \tilde{E}_b) \quad (0.2)$$

$$\tilde{I}_c = -jB(\tilde{V}_c - \tilde{E}_c) \quad (0.3)$$

$$I_{KD} = G(V_{KD} - U_{KD}) \quad (0.4)$$

$$I_{AD} = G(V_{AD} - U_{AD}) \quad (0.5)$$

The real power balancing equation is:

$$0 = (V_{KD}I_{KD} + V_{AD}\,\mathrm{I}_{AD}) - \frac{I_{AD}^2}{G} - \frac{I_{KD}^2}{G} + \mathrm{Re}\left\{\tilde{V}_a\tilde{I}_a^* + \tilde{V}_b\tilde{I}_b^* + \tilde{V}_c\tilde{I}_c^*\right\} \quad (0.6)$$

The DC-current balancing equation is:

$$0 = G\left(V_{KD} - U_{KD}\right) + G\left(V_{AD} - U_{AD}\right) \tag{0.7}$$

The relationship between DC voltage and AC voltages is:

$$0 = \sqrt{3}\left|\tilde{E}_a\right| \cdot m_a - 0.707\left(U_{KD} - U_{AD}\right) \tag{0.8}$$

The constraint equation for the modulation index is:

$$0 \le m_a \le 1 \tag{0.9}$$

Alternate Controller (V,P): DC -voltage control & power control.

$$0 = \sqrt{3}\left|\tilde{E}_a\right| - 0.707U_{dc.\max} \tag{0.10}$$

$$0 = -U_{dc.\max} \cdot m_a + V_{dc.ref} + U_{KD} - V_{KD} - U_{AD} + V_{AD} \tag{0.9}$$

$$0 = P_{ref} - \mathrm{Re}\left\{\tilde{V}_a\tilde{I}_a^* + \tilde{V}_b\tilde{I}_b^* + \tilde{V}_c\tilde{I}_c^*\right\} \tag{0.10}$$

Inverters may generate harmonics depending on their design. For example Pulse Width Modulation inverters generate very little harmonics and most of it at higher frequencies. Pulse inverters (6, 12, 18, etc. pulse inverters) generate much more harmonics and lower frequency harmonics. Table 3.2 provides typical values of generated harmonics by pulse inverters.

Table 3.2: Typical Values of Generated Harmonics (in pu)

Converter Pulses	5TH	7 TH	11 TH	13 TH	17 TH	19 TH	23 RD	25 TH	THD
6	0.175	0.110	0.045	0.029	0.015	0.010	0.009	0.008	0.215
12	0.026	0.016	0.045	0.029	0.002	0.001	0.009	0.008	0.063
18	0.026	0.016	0.007	0.004	0.015	0.010	0.001	0.001	0.037
24	0.026	0.016	0.007	0.004	0.002	0.001	0.009	0.008	0.034

3.7.3 Conventional Power Supplies

Electronic devices obtain power from the power system network typically convert AC to low voltage DC. The circuits used for this purpose can be broadly classified as conventional (also referred to as *linear*) and switched-mode. Conventional power supplies use a simple rectifier circuit typically followed by a filter capacitor and optionally a linear voltage a regulator circuit. A typical conventional power supply is illustrated in Figure 3.37. The diode bridge D1 conducts current charging the filter capacitor C1 as long as the voltage across the capacitor is lower than the instantaneous voltage of the AC source feeding the circuit. This results in the current waveform *i(t)* illustrated in Figure 3.38 (red plot trace). Note that the duration and amplitude of the current pulses as well as the capacitor voltage ripple depend on the filter capacitance value and the load current. Increasing the capacitor size results in lower capacitor voltage ripple, but also shortens the input current pulse width and increases their amplitude, thus increasing also the input current harmonic content.

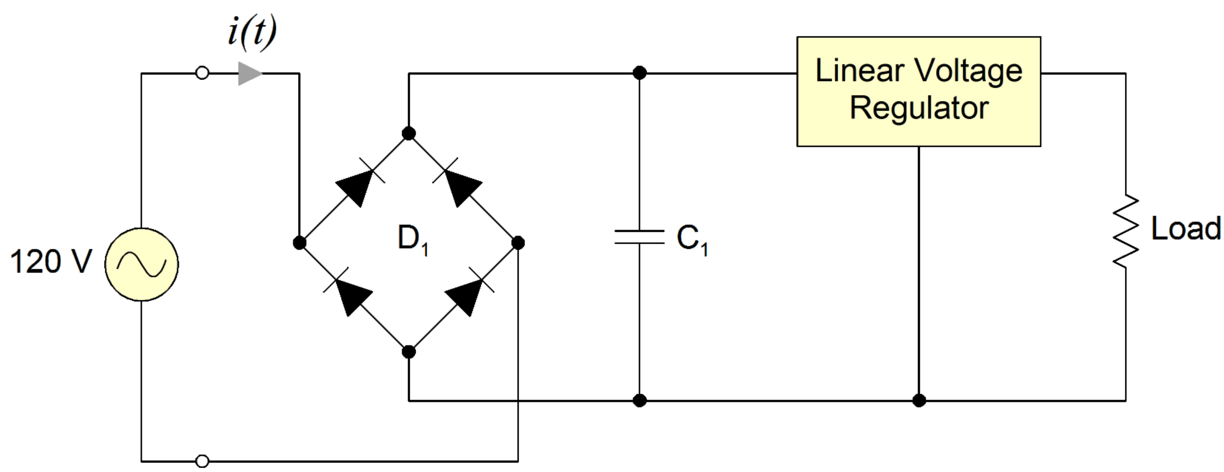


Figure 3.37: Conventional Power Supply Circuit Diagram

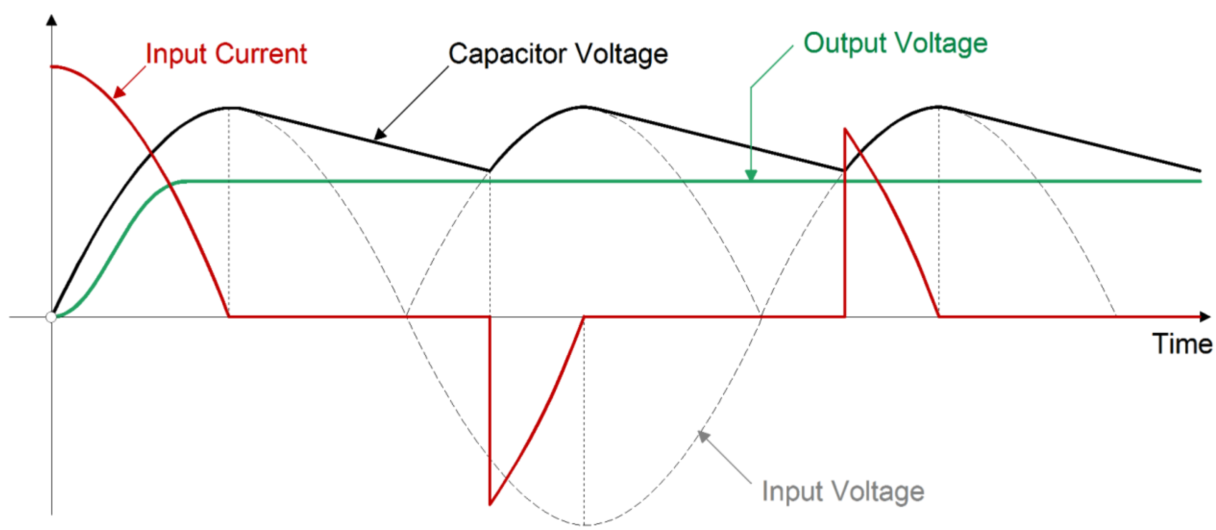


Figure 3.38: Conventional Power Supply Voltage and Current Waveforms

In order to generate a constant output voltage conventional power supplies often include a linear voltage regulator. The linear voltage regulator typically contains a series power transistor that adds significant power loss. Specifically the ripple voltage (i.e. the difference between the capacitor voltage and the desired constant output voltage) appears across the transistor collector-emitter terminals while the current trough the transistor is equal to the load current. Thus the power loss in the transistor is equal to the product of the ripple voltage multiplied by the load current.

Modern electronic devices often include switch-mode power supplies in which the linear regulator is replaced by a switching regulator. Switching regulators control the output voltage by high frequency switching schemes and can be significantly more efficient and also more compact than linear regulators. However, switch-mode power supplies may generate high frequency electromagnetic noise, causing radio interference.

3.7.4 Fluorescent Lighting

Florescent lights provide higher efficiency than incandescent lights and thus they have been widely used in commercial and industrial applications. Conventional florescent consist of a glass tube containing mercury vapor. Two electrodes located at the ends of the tube initiate a discharge arc through the mercury vapor which produces ultraviolet light. A phosphor coating inside the glass tube converts the ultraviolet light to visible light. The electric arc is initiated and extinguished every half cycle resulting in highly distorted current waveform. A typical florescent light current waveform is illustrated in Figure 3.39.

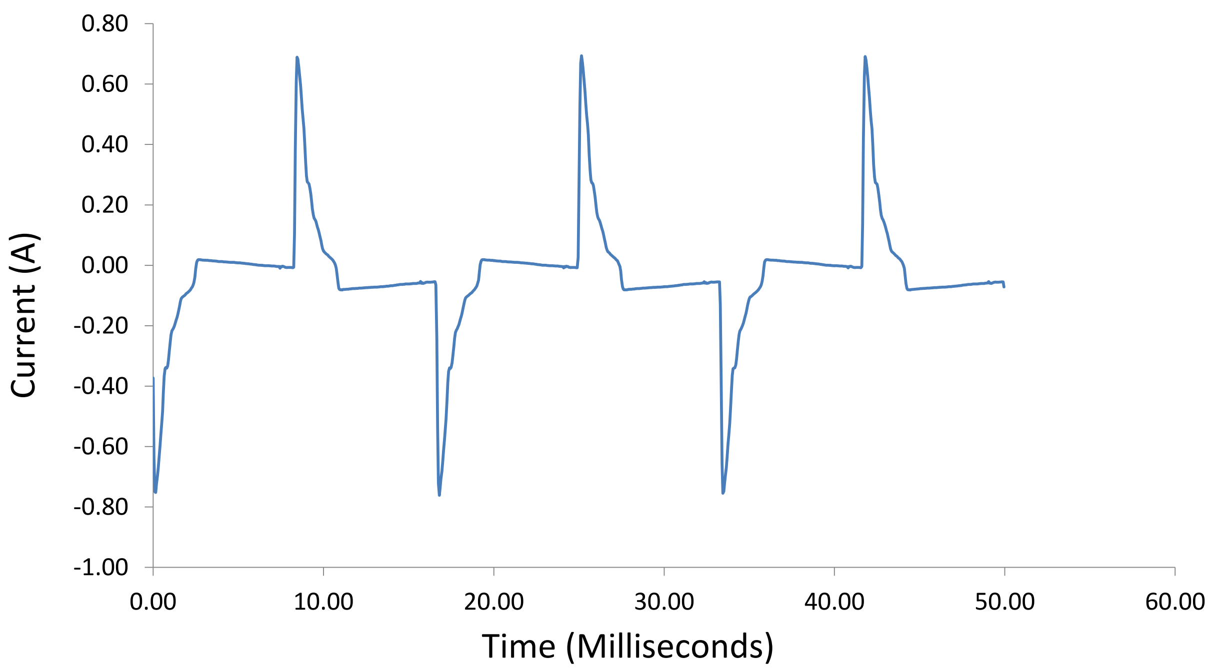


Figure 3.39: Fluorescent Light Current Waveform

The initiation of the electric arc in a fluorescent light requires a relatively high voltage. Furthermore, once the arc is initiated the voltage across the arc must be reduced in order to limit the current. Thus, fluorescent lights require additional hardware that provides these functions known as the *ballast circuit*.

A fluorescent light with a conventional ballast circuit is illustrated in Figure 3.40. The circuit contains a starter switch consisting of a bimetallic switch enclosed in a neon gas tube. When the power is first applied to the circuit the starter switch is closed. This allows electric current to flow through the heating filaments which warm up the mercury into vapor. After a short delay the bimetallic element temperature rises causing the starter switch to open. The current flowing through the ballast inductor is interrupted causing a high voltage transient across the two heater filaments igniting an electric arc through the mercury vapor. Note that the ballast inductor is in series with the mercury arc, providing current limiting impedance.

Conventional ballast circuits have been displaced by “electronic ballasts” which have several advantages over conventional ballasts:

- Provide more efficient operation
- Eliminate fluorescent light flicker
- Prolong the life of fluorescent lights
- Are lighter and more compact than conventional ballasts

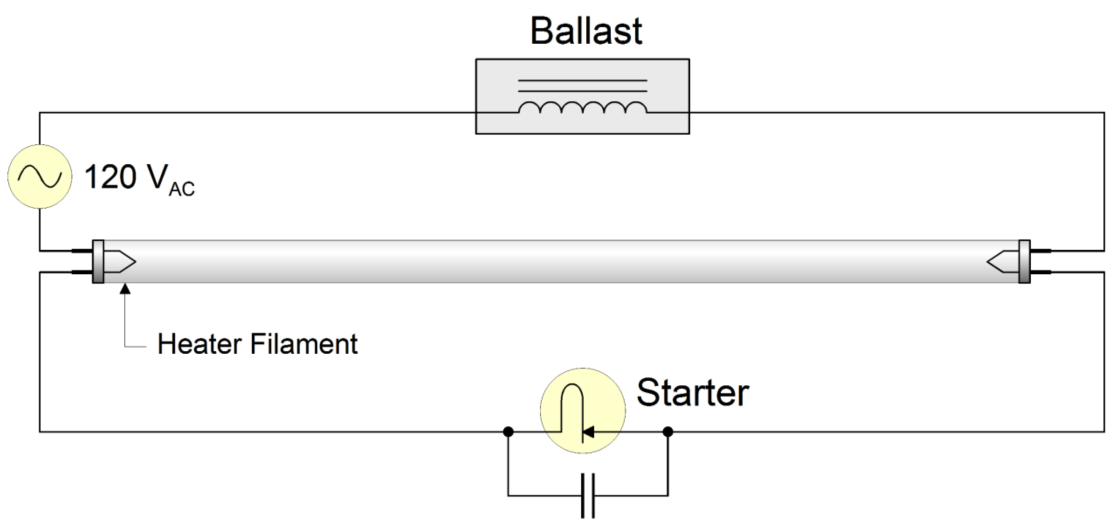


Figure 3.40: Fluorescent Light Conventional Ballast Circuit

A basic electronic ballast circuit is illustrated in Figure 3.41. It consists of back-to-back rectifier and inverter circuits. The inverter typically operates at frequencies in the order of 10 to 20 kHz. The high frequency applied across the fluorescent light electrodes provides a continuous arc thus eliminating any flicker and also increasing light output efficiency.

A variety of electronic ballasts are presently available. Some implementations include a microprocessor based controller which provides alternative starting methods which do not employ the heating filaments. They may also adjust the applied frequency and voltage for optimal operation under various temperatures thus prolonging the lamp life. Furthermore microprocessor based implementations may perform diagnostics that detect lamp failures lamp presence etc. Additionally, advanced implementations may include circuitry that reduce input current harmonics and provide near unity power factor.

Recently compact fluorescent and light emitting diode (L.E.D.) based technologies have been developed yielding devices which directly replace incandescent light bulbs. These devices include electronic interface circuitry within the light bulb base. Note that L.E.D. based lights provide even higher efficiency and longer life than compact fluorescent lights. Both compact fluorescent and LED based lights obtain power from the AC power network through a rectifier. Thus all such devices draw distorted current waveforms and thus generate various levels of current harmonics. Figure 3.42 illustrates typical current waveforms of two types of LED based lights.

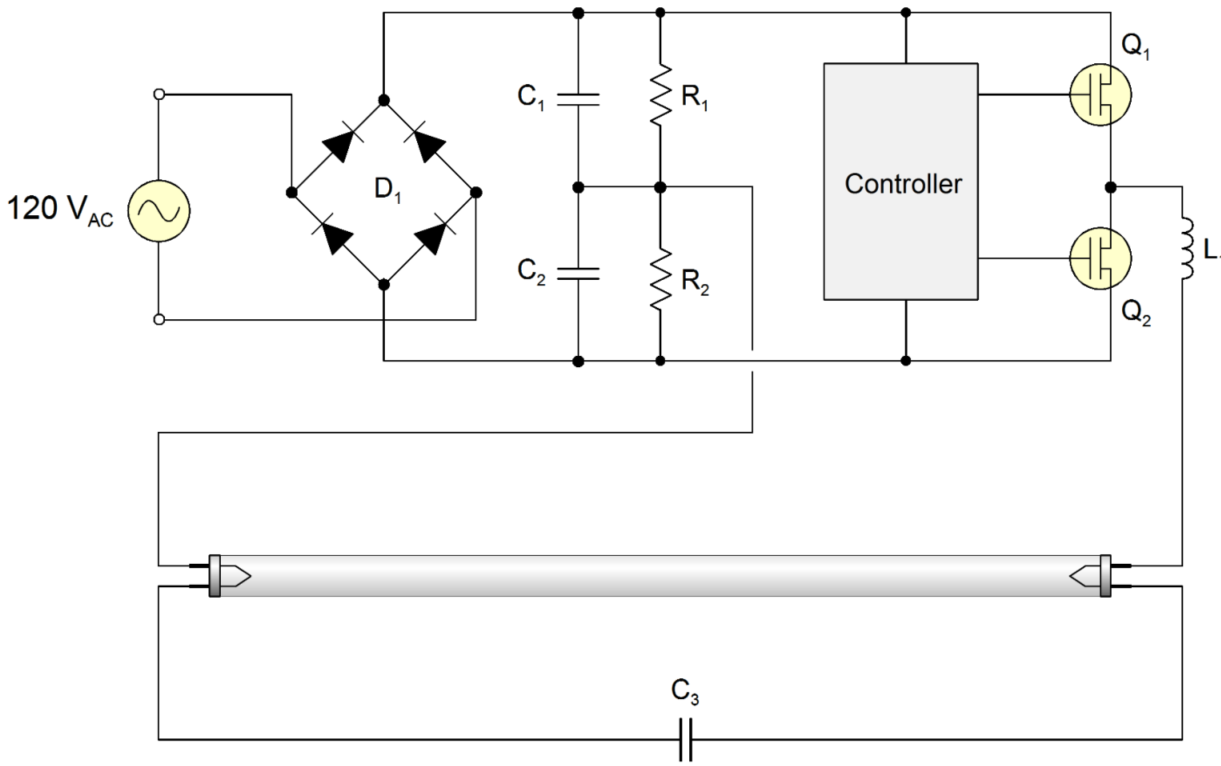
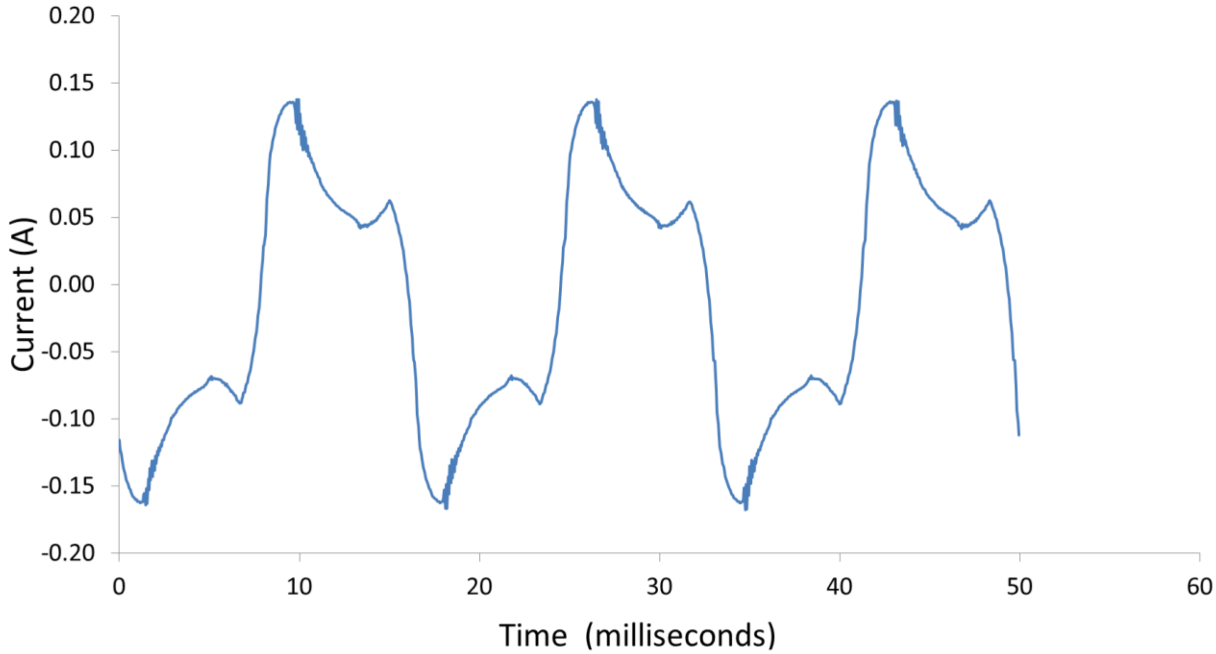
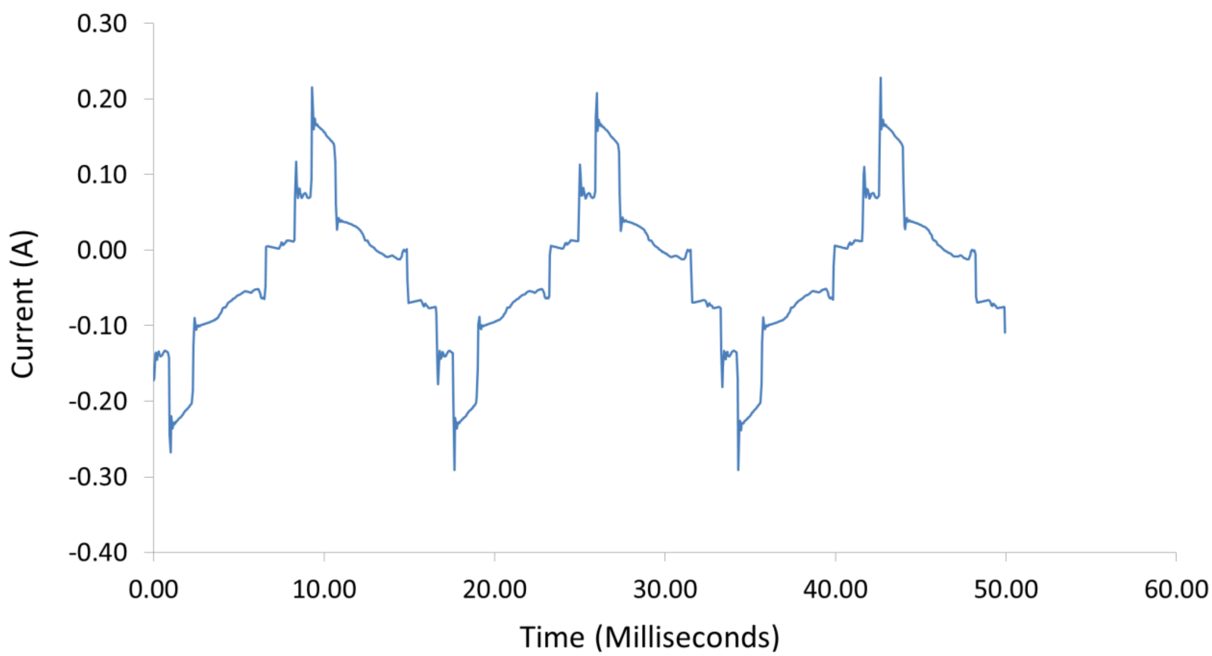


Figure 3.41: Fluorescent Light Electronic Ballast Circuit



(a)



(b)

Figure 3.42: Light Emitting Diode Based Lamp Current Waveforms
(a) Non-Dimmable Type, (b) Dimmable Type

3.7.5 Dimmers

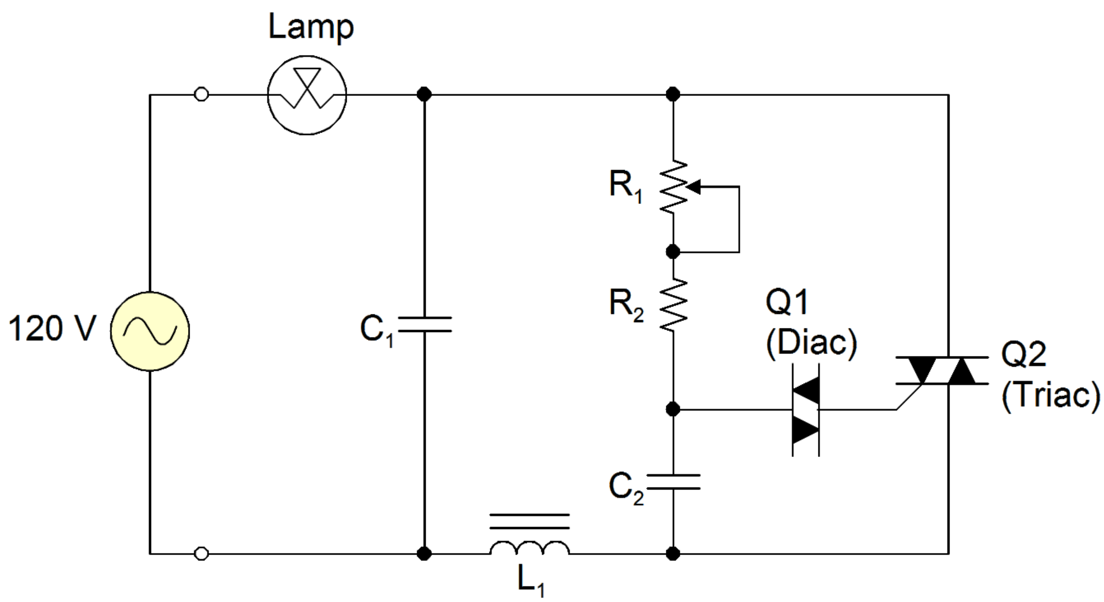
There is a large array of products providing appliance power level control using power electronics. Specifically, these devices use repetitive switching to control the power output of various appliances, such as fans, heaters, lighting fixtures, etc. Typically, the power is switched on and off every half cycle so that the device is connected to the power supply for only a certain portion of each half cycle. We refer to these devices as dimmers since their main application is in controlling the light output of lighting devices.

A typical incandescent light dimmer circuit and the resulting electric current waveform is illustrated in Figure 3.43. The circuit contains a triac (Q2) which switches on after a certain delay from the waveform zero crossing. The delay time is controlled the potentiometer R_1. By increasing the delay angle, the RMS value of the current is decreased. Depending on the selected delay the current waveform will contain certain harmonic levels, with the total harmonic distortion generally increasing with the delay angle. The capacitor C1 and the inductor L1 provide a filtering function which reduces the harmonic levels. The current harmonic levels for several delay angles, neglecting the effect of the harmonic filter are shown in Table 3.3.

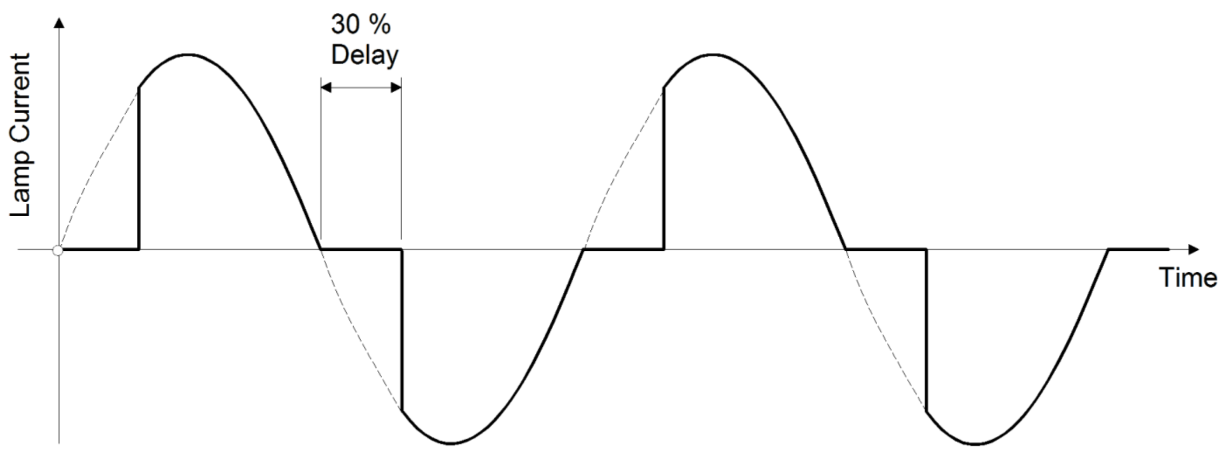
Depending on the quality of the filter design such simple dimmer circuits may generate significant harmonic levels and may result in radio and TV interference. More advanced dimmer circuits are available (at a higher cost) that generate very low harmonic levels, based on high frequency switching and pulse width modulation techniques.

Table 3.3: Typical Harmonics Generated By Light Dimmers

Delay Angle	1st (%)	3rd (%)	5th (%)	7th (%)	9th (%)	11th (%)	13th (%)	15th (%)
10	1.00	0.83	0.82	0.81	0.79	0.77	0.74	0.71
30	1.00	8.00	7.10	5.89	4.57	3.32	2.37	1.91
50	1.00	22.6	15.8	9.10	5.75	5.60	5.06	3.86
70	1.00	46.9	22.1	13.0	12.6	8.58	7.84	7.02



(a)



(b)

Figure 3.43: Light Dimmer Example

(a) Circuit Diagram, (b) Current Waveform

3.8 Application Examples

This section presents few application examples of the models described in this chapter.

3.8.1 Inrush Currents During Transformer Energization

To be added.

3.8.2 Transformer Performance during Direct Current Flow in Neutral (Geomagnetically Induced)

To be added.

3.8.3 Harmonic Currents in Converter Transformers

To be added.

3.8.4 Induced Voltage on Parallel Conductors/Power Lines

Conductors placed parallel to power lines are subject to induced voltages. The level of the voltages can be computed with the models developed so far. Specifically, the voltage on a conductor parallel to a power line (telephone wire, fence, etc.) is given by

$$\tilde{V}_t = \sum_k z_{tk} \tilde{I}_k$$

The mutual impedance z_{tk} can be computed with any of the three models presented earlier. The procedure will be demonstrated with an example.

Example E3.8: Consider a three phase 25 kV overhead transmission line. A telephone line (wire pair) parallels the power line for a distance of one mile. The relative position of the power line and telephone line is illustrated in Figure E3.8. The power line carries the following electric currents.

$$i_a(t) = \sqrt{2}200\cos\omega t + \sqrt{2}50\cos(5\omega t + 10^0) + \sqrt{2}40\cos(7\omega t + 30^0)$$
$$i_b(t) = \sqrt{2}200\cos(\omega t - 120^0)$$
$$i_c(t) = \sqrt{2}200\cos(\omega t - 240^0) + \sqrt{2}50\cos(5\omega t + 250^0) + \sqrt{2}40\cos(7\omega t - 210^0)$$

$$\omega = 2\pi 60\sec^{-1}$$

(a) Compute the electric current in the neutral (for simplicity assume that the electric current in the earth is zero).
(b) Compute the induced voltage on the telephone line per unit length.

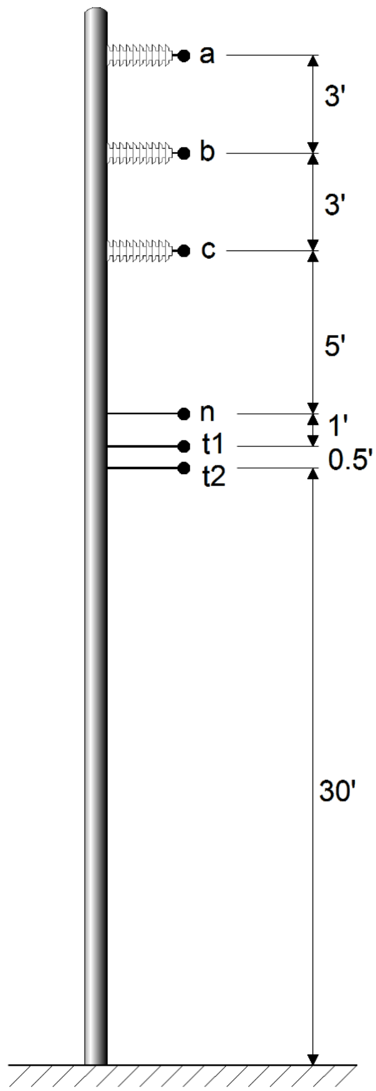


Figure E3.8: A Communication Line Suspended on a Power Pole

Solution: (a) the electric current in the neutral will be the negative sum of all the currents:

$$i_n(t) = -i_a(t) - i_b(t) - i_c(t) = \sqrt{2}50\cos(5\omega t + 130^0) + \sqrt{2}40\cos(7\omega t - 90^0)$$

(b) The induced voltage on the telephone circuit is computed separately for each harmonic.

Fundamental:

$$\tilde{V}_{t1} - \tilde{V}_{t2} = \frac{j\omega\mu}{2\pi}\left[\ln\left(\frac{12.0}{12.5}\right)200 + \ln\left(\frac{9.0}{9.5}\right)200e^{-j120^0} + \ln\left(\frac{6.0}{6.5}\right)200e^{-j240^0} \right] = 0.521e^{-j40.6^0} \ mV/m$$

5th Harmonic:

$$\tilde{V}_{t1} - \tilde{V}_{t2} = \frac{j5\omega\mu}{2\pi}\left[\ln\left(\frac{12.0}{12.5}\right)50e^{j10^0} + \ln\left(\frac{6.0}{6.5}\right)50e^{-j250^0} + \ln\left(\frac{1.0}{1.5}\right)50e^{-j130^0} \right] = 6.535e^{-j66.35^0} \ mV/m$$

7th Harmonic:

$$\tilde{V}_{t1} - \tilde{V}_{t2} = \frac{j7\omega\mu}{2\pi}\left[\ln\left(\frac{12.0}{12.5}\right)40e^{j30^0} + \ln\left(\frac{6.0}{6.5}\right)40e^{-j210^0} + \ln\left(\frac{1.0}{1.5}\right)40e^{-j90^0} \right] = 7.319e^{-j84.4^0} \ mV/m$$

3.9 Problems

Problem P3.1: Consider the three-phase overhead transmission line illustrated in Figure P3.1. The line is constructed in an area that has soil resistivity equal to 185 ohm.meters. The phase conductors are ACSR, BITTERN and the shield wires are ALUMOWELD, 3#7AW. The resistance, geometric mean radius and diameter of these conductors can be obtained from tables and they are provided below at 60 Hz.

ACSR, BITTERN: $r = 0.0729\, ohms/mile, \quad GMR = 0.04447\ feet, \quad d = 1.345\, inches$
ALUMOWELD, 3#7AW: $r = 4.420\, ohms/mile, \quad GMR = 0.002351\, feet, \quad d = 0.311\, inches$

(a) For simplicity, neglect the shield wires and compute the positive, negative and zero sequence parameters of the line per unit length. In other words compute the following parameters: $pos\ seq:(R_1, L_1, C_1), \quad neg\ seq:(R_2, L_2, C_2), \quad and\ zero\ seq:(R_0, L_0, C_0)$.
Your answer should be in ohms per meter, henries per meter and farads per meter.
(b) Using the parameters from (a) compute the nominal pi-equivalent positive, negative and zero sequence circuits at 60 Hz. The total line length is 94.5 miles.
(c) Use the computer program WinIGS to model this line and compute the positive, negative and zero sequence pi-equivalent circuit of the line. Compare the computer results to your results in part (b).

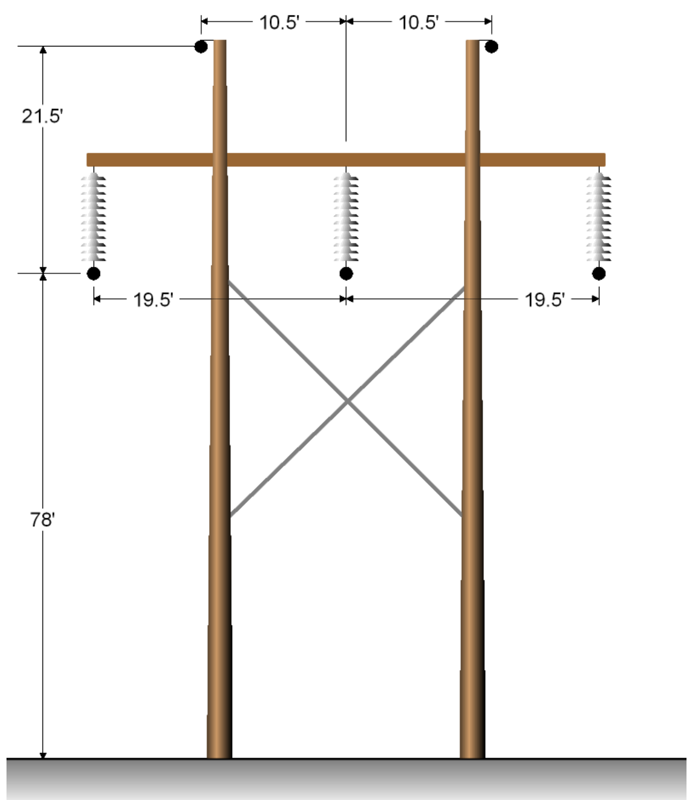


Figure P3.1

Solution: (a) First we compute the R, L and C matrices:

r_c = 0.0729 Ω/mi = 45.3 μΩ/m, r_e = 0.00159f / 1609 = 59.29 μΩ/m

$$R = \begin{bmatrix} 104.59 & 59.29 & 59.29 \\ 59.29 & 104.59 & 59.29 \\ 59.29 & 59.29 & 104.59 \end{bmatrix} \times 10^{-6} \Omega/m$$

$$D_e = 2160 \sqrt{\frac{\rho}{f}} \text{ ft} = 3{,}792.84 \text{ ft}$$

$$L_{ij} = \frac{\mu}{2\pi} ln \frac{D_e}{d_{ij}}$$

where d_{ij} = distance i – j, and d_{ii} = GMR_i

$$L = 2\begin{bmatrix} 11.354 & 5.27 & 4.577 \\ 5.27 & 11.354 & 5.27 \\ 4.577 & 5.27 & 11.354 \end{bmatrix} \times 10^{-7} \text{H}/m$$

→ L_s = 2.27 μH/m, L_m = 1.007 μH/m

$$C'_{ij} = \frac{1}{2\pi\varepsilon} ln \frac{d_{ij'}}{d_{ij}}$$

$$C' = \frac{1}{2\pi\epsilon}\begin{bmatrix} 7.931 & 2.087 & 1.416 \\ 2.087 & 7.931 & 2.087 \\ 1.416 & 2.087 & 7.931 \end{bmatrix}$$

$$C = C'^{-1} = 2\pi\epsilon \begin{bmatrix} 0.1373 & -0.0319 & -0.0161 \\ -0.0319 & 0.1428 & -0.0319 \\ -0.0161 & -0.0319 & 0.1373 \end{bmatrix}$$

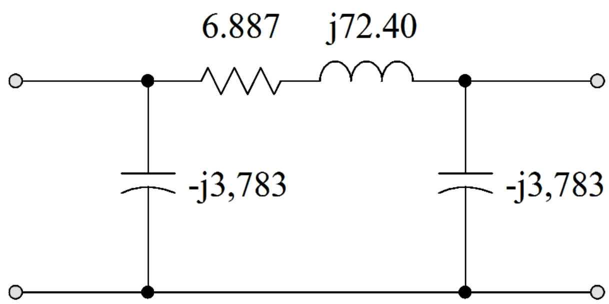
→ C_s = 7.74 pF/m, C_m = -1.481 pF/m

→ $R_1 = R_2 = 45.3 \times 10^{-6}$ Ω/m, $L_1 = L_2 = 1.263 \times 10^{-6}$ H/m, $C_1 = C_2$ = 9.221 pF/m

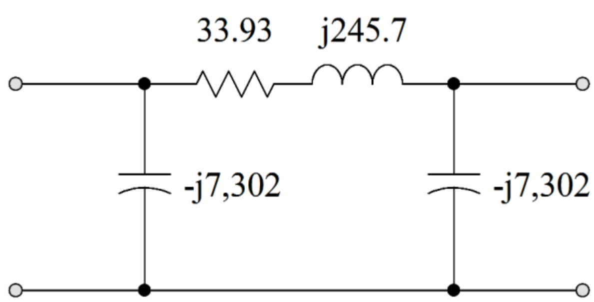
→ $R_0 = 223.17 \times 10^{-6}$ Ω/m, $L_0 = 4.2864 \times 10^{-6}$ H/m, C_0 = 4.778 pF/m

(b)

Positive or Negative Sequence Network



Zero Sequence Network



Problem P3.2: The wires of a fence "run" parallel with a power line as it is illustrated in Figure P3.2. During a certain phase to ground fault, the electric currents in the power line are as follows:

phase A: $\tilde{I}_a$ = negligible

phase B: $\tilde{I}_b$ = negligible

phase C: $\tilde{I}_c$ = 10,000 A

(a) Compute the earth current.

(b) Compute the induced voltage in volts per meter on the fence wire F1. The soil resistivity is 100 Ω·meter.

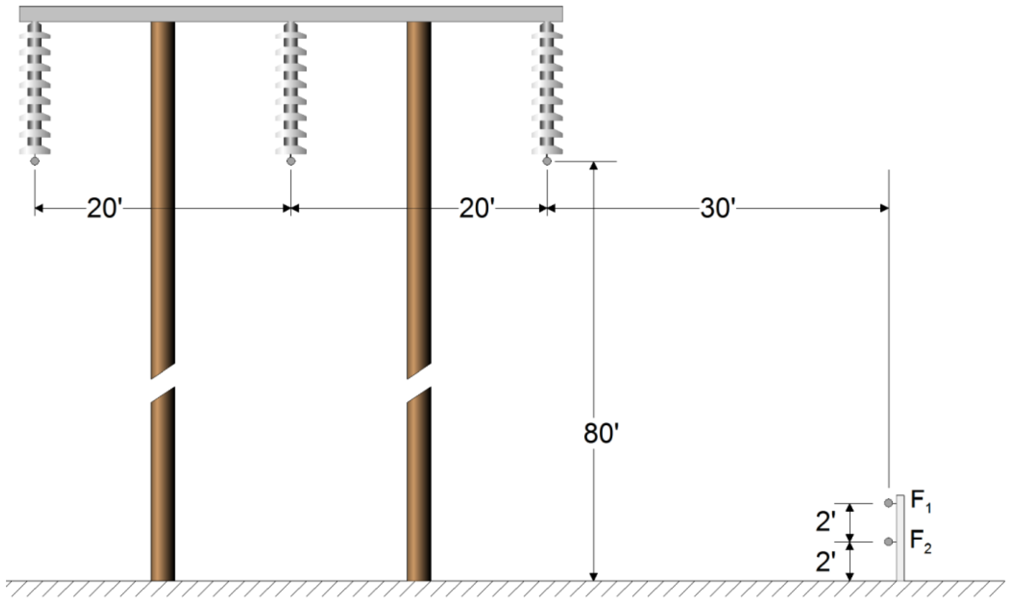


Figure P3.2

Solution:

(a) $\tilde{I}_e = -(\tilde{I}_a + \tilde{I}_b + \tilde{I}_c) = -10kA$

(b) $\tilde{V}_{F1} = j\dfrac{\mu\omega}{2\pi}\tilde{I}_c \ln\dfrac{D_e}{D_{F1c}}$

where

$$D_e = 2160\sqrt{\frac{100}{60}} = 2788.548\ ft$$

$$D_{F1c} = \sqrt{30^2 + 76^2} = 81.7\ ft$$

$$V_{F1c} = 377 \times 2 \times 10^{-7} \times 10^4 \times \ln\frac{2788.548}{81.7} = 2.66\,V/m$$

Problem P3.3: Consider the three-phase, 60 Hz, transmission line of the Figure P3.3. The phase conductors are ACSR with the following parameters:

Radius:	0.85 inches
Geometric Mean Radius:	0.059 feet

The ground wire is neglected. The soil resistivity is 135 ohm.meters. Compute the positive sequence series inductance in <u>Henries per meter.</u>

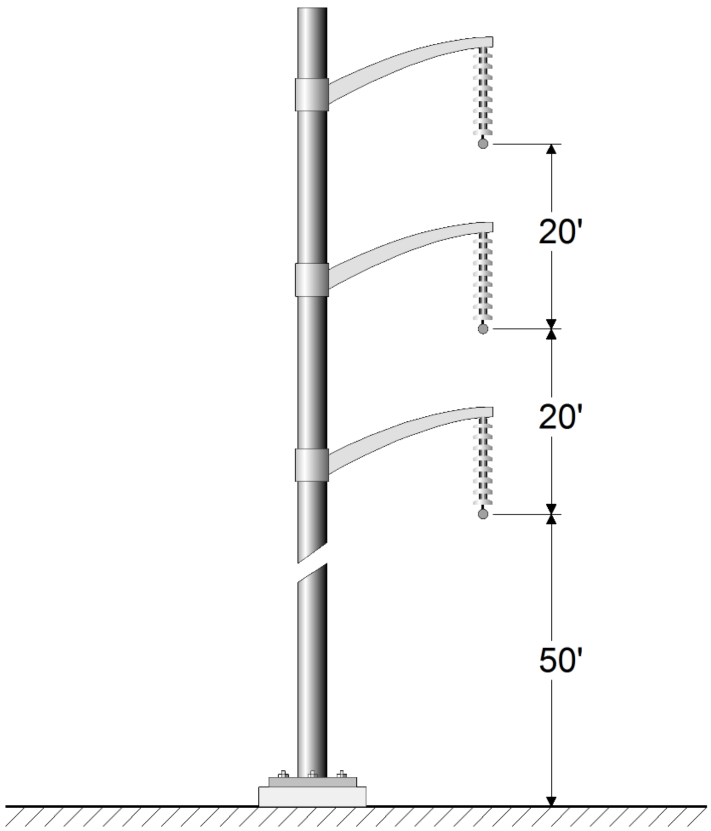


Figure P3.3

Solution: First the inductance matrix is computed:

$$L = \frac{\mu}{2\pi}\begin{bmatrix} ln\dfrac{D_e}{d} & ln\dfrac{D_e}{d_{ab}} & ln\dfrac{D_e}{d_{ac}} \\ ln\dfrac{D_e}{d_{ab}} & ln\dfrac{D_e}{d} & ln\dfrac{D_e}{d_{bc}} \\ ln\dfrac{D_e}{d_{ac}} & ln\dfrac{D_e}{d_{bc}} & ln\dfrac{D_e}{d} \end{bmatrix}$$

$D_e = 2160\sqrt{\frac{\rho}{f}} = 3240$, d = 0.059 ft, d_{ab} = 20 ft, d_{ac} = 40 ft, d_{bc} = 20ft

$$\Rightarrow \quad L = 2\times 10^{-7}\begin{bmatrix} 10.9135 & 5.0876 & 4.3944 \\ 5.0876 & 10.9135 & 5.0876 \\ 4.3944 & 5.0876 & 10.9135 \end{bmatrix}$$

$$\Rightarrow \; L_1 = L_s - L_m = 1.211\ \mu H / m$$

Problem P3.4: Consider the three-phase overhead transmission line illustrated in Figure P3.4. For simplicity, assume that the phase conductors of the line are solid aluminum conductors with diameter 1.0 inch and conductivity 40,000,000 S/m.

(a) Compute the pi-equivalent positive, negative and zero sequence circuits for 60 Hz, 180 Hz and 540 Hz. The line length is 56.5 miles. Neglect the ground wires. The soil resistivity is 225 ohm-meters.

(b) Use the computer program WinIGS to model this line and compute the positive, negative and zero sequence pi-equivalent circuit of the line for 60 Hz, 180 Hz and 540 Hz.

The shield wires are ALUMOWELD, Compare the computer results to your results in part (a). Note that one procedure neglects the shield wires and the other does not.

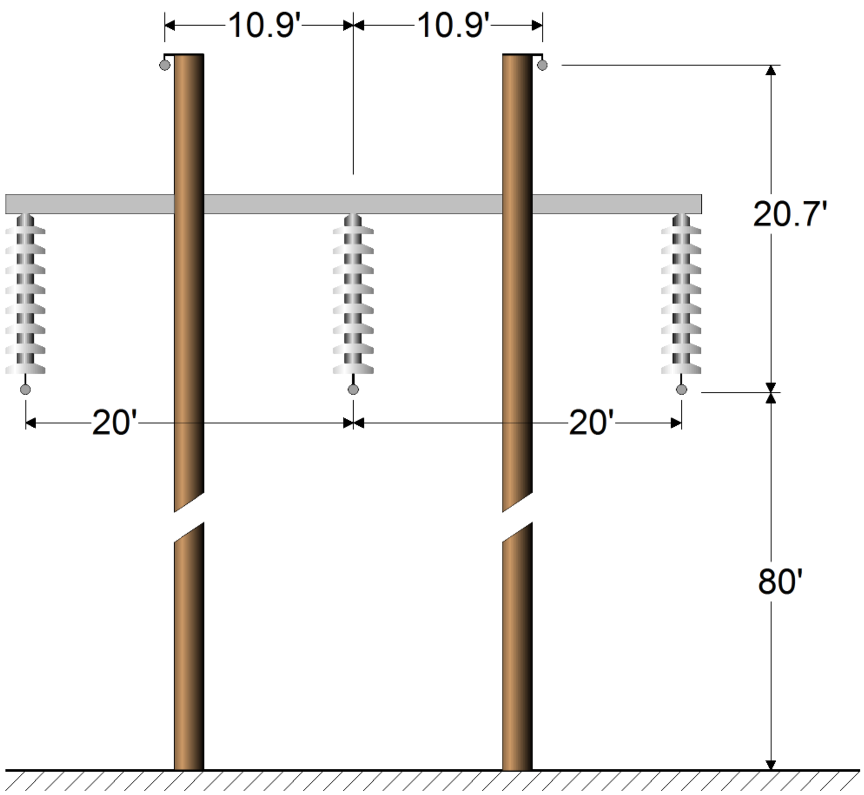


Figure P3.4

Problem P3.5: Consider a three phase, 480 V (line to line) power line feeding a rectifier. Assume that the rectifier generates (a) third harmonic currents which are 42% of the fundamental and zero sequence, (b) fifth harmonic currents which are 38% of the fundamental and positive sequence, and (c) seventh harmonic currents which are 28% of the fundamental and negative sequence. The power line consists of four 1.5 inch diameter solid copper conductors: three phase conductors and one neutral conductor.

(a) Compute the resistance ratio r_{ac}/r_{dc} for the third, fifth and seventh harmonics.

(b) Compute the ohmic losses of this circuit.

The copper conductivity is 57,000,000 S/m

Problem P3.6: Compute the inductance and resistance of a 1,000,000 cm solid copper conductor located 10 meters above 100 ohm.meter soil in the frequency range 1.0 to 420 Hz. Assume that the conductor carries the following currents which return though the soil:

500 A of 60 Hz
50 A of 300 Hz, and
50 A of 420 Hz.

Compute the total ohmic losses in watts per meter. The conductivity of the copper conductor is σ=54,000,000 S/m

Problem P3.7: Consider a 277 V power line (single phase) feeding a rectifier. Assume that the rectifier generates third harmonic currents which are 58% of the fundamental. The power line consists of two 2-inch diameter copper conductors: one phase conductor and one neutral conductor. The line is 300 meters long.

At the present operating condition, the fundamental frequency current of the rectifier is 850 Amperes.

Compute the Ohmic losses in the entire length (300 meters) of the power line.

The resistivity of copper is: $1.8\text{x}10^{-8}$ ohm-meters.

Problem P3.8: Consider the three-phase overhead distribution line illustrated in Figure P3.8. The phase conductors of the line are solid aluminum conductors with diameter 1.0 inch. The line length is 5 miles. The soil resistivity is 225 ohm-meters.

(a) Compute the zero sequence pi-equivalent circuit for the 11^{th} harmonic.
(b) Compute the positive, negative and zero sequence pi-equivalent circuit for the 9^{th} harmonic.

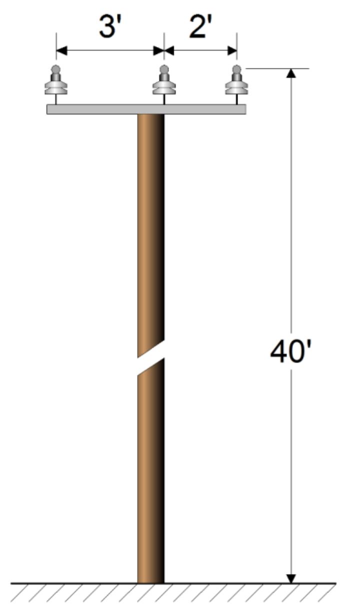


Figure P3.8

The resistivity of aluminum is: $2.8\text{x}10^{-8}$ ohm-meters

Problem P3.9: The two wires of a pilot relaying scheme "run" parallel with the power line as it is illustrated in Figure P3.9. During a certain phase to ground fault, the electric currents in the power line are as follows:

phase A: $\tilde{I}_a$ = negligible

phase B: $\tilde{I}_b$ = negligible

phase C: $\tilde{I}_c$ = 12,500 A

Compute the induced voltage in volts per meter on the pilot wire P1. If the line is 1.8 miles long, what is the total voltage induced on P1?

Compute the induced voltage between the wires P1 and P2. (i.e. compute the induced voltage on wire P1 and P2 an then take the difference).

The soil resistivity is 185 ohm-meters. For simplicity neglect the shield wires.

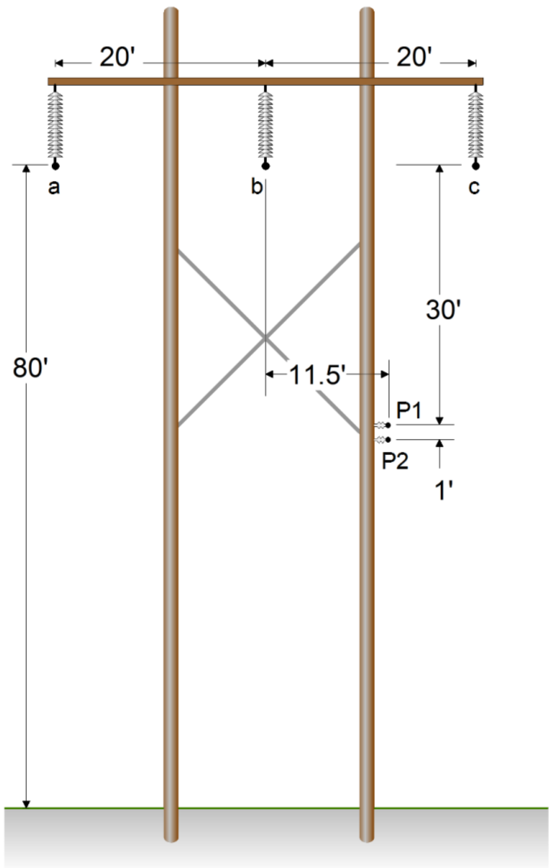


Figure P3.9

Solution:

$$V = \frac{j\omega\mu}{2\pi}\ I\ ln\frac{D_e}{d_{ab}}$$

$$D_e = 2160\sqrt{\frac{\rho}{f}} = 3{,}792.8\ ft$$

$$d_{ab} = \sqrt{30^2 + 8.5^2 = 31.18\ ft}$$

$\rightarrow$ V = 4.525 V/m

1.8 miles = (1.8) (1609 m/mi) = 2,896.2 m

$\rightarrow$ V = (4.525) (2,896.2) V = 13,105 Volts